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Duplication of a Vertex by a new Vertex in Divided Square Difference Cordial Graphs

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Abstract

In this present work, we discuss divided square difference (DSD) cordial labeling in the context of duplication of a vertex by a new vertex in DSD cordial graphs such as path graph, cycle graph, star graph, wheel graph, helm graph, crown graph and comb graph.

Keywords:— *duplication of a vertex by a vertex, path graph, cycle graph, star graph, wheel graph, helm graph, crown graph, comb graph.*

AMS Subject Classification (2010): *05C78.*

I. Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges. For standard terms we refer to Harary [1]. In 1967, Rosa [2] introduced a labeling of *G* called β-valuation. A dynamic survey on different graph labeling along with an extensive bibliography was found in Gallian [3]. The concept of cordial labeling was introduced by Cahit [4]. R. Varatharajan, et.al [5] have introduced the notion of divisor cordial labeling. A. Alfred Leo et.al [6] introduced the concept of divided square difference cordial labeling graphs. Also discussed DSD cordial labeling for more graphs in [7] and [8]. V. J. Kaneria et.al [9] introduced the concept of balanced cordial labeling. From that A. Alfred Leo et.al [10] defines the concept of balanced divided square difference cordial labeling. The motivation behind the DSD cordial labeling is due to R. Dhavaseelan et.al on their work even sum cordial labeling graphs [11].

The motivation behind this work is due to S. K. Vaidya et.al on their work Harmonic mean labeling in the context of duplication of graph elements [12]. In this present work, we discuss divided square difference (DSD) cordial labeling in the context of duplication of a vertex by a new vertex in DSD cordial graphs such as path graph, cycle graph, star graph, wheel graph, helm graph, bistar graph, crown graph and comb graph.

II. Preliminaries

Definition 2.1 [3]

The G*raph labeling* is an assignment of numbers to the edges or vertices or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges), then the labeling is called a vertex (edge) labeling.

Definition 2.2 [3]

A *vertex labeling* of a graph *G* is an assignment *f* of labels to the vertices of *G* that induces each edge uv a label depending on the vertex label $f(u)$ and $f(v)$.

Definition 2.3 [3]

A mapping $f:V(G)\rightarrow\{0,1\}$ is called *binary vertex labeling* of G and $f(V)$ is called the label of the vertex v of G under f.

Definition 2.4 [4]

A binary vertex labeling f of a graph G is called a *Cordial labeling* if $|v_f(0)-v_f(1)|\leq 1$ and if $|e_f(0)-e_f(1)|\leq 1$. A graph G is cordial if it admits cordial labeling.

Definition 2.5 [1]

The *wheel graph* W_n is a graph formed by connecting a single universal vertex to all vertices of a cycle.

Definition 2.6 [1]

The *helm graph* H_n is the graph obtained from a wheel graph by adjoining a pendent edge at each node of the cycle.

Definition 2.7 [1]

The *corona* $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 (with p_1 vertices) and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to all the vertices in the i^{th} copy of G_2.

The graph $P_n \odot K_1$ is called a *comb*.

The graph $C_n \odot K_1$ is called a *crown*.

Definition 2.8 [12]

Duplication of a vertex v_k of a graph G produces a new graph G_D by adding a vertex v_k' with $N(v_k)=N(v_k')$.

In other words, a vertex v_k' is said to be duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v_k' also.

Definition 2.9 [9]

A cordial graph G with a cordial labeling f is called a *balanced cordial graph* if.

$|e_f(0)-e_f(1)| = |v_f(0)-v_f(1)| = 0.$

It is said to be *edge balanced cordial graph* if

$|e_f(0)-e_f(1)|=0$ and $|v_f(0)-v_f(1)|=1.$

It is said to be *vertex balanced cordial graph* if $|e_f(0)-e_f(1)| = 0$ and $|v_f(0)-v_f(1)|=0$.

A cordial graph G is said to be *unbalanced cordial graph* if $|e_f(0)-e_f(1)|=|v_f(0)-v_f(1)|=1$.

Definition 2.10 [6]

Let $G=(V,E)$ be a simple graph and $f:V\rightarrow\{1,2,3,\dots|V|\}$ be a bijection. For each edge, assign the label if

$$\left|\frac{(f(u))^2-(f(v))^2}{f(u)-f(v)}\right|$$

is odd and the label 0 otherwise. f is called divided square difference cordial labeling if where $|e_f(0)-e_f(1)|\leq 1$, $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively.

A graph G is called divided square difference cordial if it admits divided square difference cordial labeling.

Definition 2.11 [10]

A divided square difference cordial graph G is called a *balanced divided square difference cordial graph* if $|e_f(0)-e_f(1)|=0$.

A divided square difference cordial graph G is called a *unbalanced divided square difference cordial graph* if $|e_f(0)-e_f(1)|=1$.

Proposition 2.12 [6]

1. Any path p_n is a divided square difference cordial graph.

2. Any cycle C_n is a divided square difference cordial graph except. $n \equiv 2$ (mod 4).
3. The star graph $K_{1,n}$ is a divided square difference cordial.

Proposition 2.13 [7]

1. The wheel graph $W_n, (n \equiv 0,1 \ (mod\ 4))$ is a divided square difference cordial.
2. The helm graph $H_n, (n \equiv 0,1 \ (mod\ 4))$ is a divided square difference cordial.

Proposition 2.14 [8]

1. The crown graph $C_n \odot K_1$ is a divided square difference cordial.
2. The comb graph $P_n \odot K_1$ is a divided square difference cordial.

Note 2.15

In the new graph G_D, the new vertex v_k' can be labeled as $f(v_k') = N$ where $N = |V(G_D)|$.

III. Results and Discussion

Proposition 3.1

A graph obtained by duplication of a vertex v_k by a new vertex in a DSD cordial path P_n is DSD cordial.

Proof

Let G be a path graph P_n Let $v_1, v_2, \ldots, v_n$ are the vertices of the path P_n In this graph $|V(G)| = n$ and $|E(G)| = n-1$. By Proposition 2.12, we draw a DSD cordial path P_n.

Case i: Duplicating a pendent vertex (except) *$n \equiv 0 \ (mod\ 4)$)*

Now, without loss of generality we duplicate any of the pendent vertex v_k in G by a vertex v_k' and construct a new graph G_D. In this graph G_D, $|V(G_D)| = n+1$ and $|E(G_D)| = n$. For DSD cordial labeling pattern, let the vertex labels are $\{1,2,\ldots,n+1\}$. Then, by labeling the graph G_D using definition 2.10 and the new vertex v_k' by $f(v_k') = n+1$, we get $|e_f(0) - e_f(1)| \leq 1$.

Hence G_D is also a divided square difference cordial graph.

Case ii: Duplicating a vertex of degree 2

Now, without loss of generality we duplicate any arbitrary vertex v_k of degree 2 in G (except $n-1^{th}$ vertex for $n \equiv 2 (mod\ 4)$) by a vertex v_k' and construct a new graph G_D In this graph, G_D, $|V(G_D)| = n+1$ and $|E(G_D)| = n+1$. For DSD cordial labeling pattern, let the vertex labels are $\{1,2,\ldots,n+1\}$. Then, by labeling the graph G_D using definition 2.10 and the new vertex v_k' by, $f(v_k') = n+1$, we get $|e_f(0) - e_f(1)| \leq 1$.

Hence G_D is also a divided square difference cordial graph.

Remark 3.2

From Proposition 3.1–case-i, we get

$$|e_f(0) - e_f(1)| = \begin{cases} 1, n \text{ is odd} \\ 0, n \text{ is even} \end{cases}$$

Hence, we can conclude that G_D (when duplicate a pendent vertex v_k) is a balanced DSD cordial graph when n is even and unbalanced DSD cordial when n is odd.

Similarly from Proposition 3.1–case-ii, in particular we get

$$|e_f(0) - e_f(1)| = \begin{cases} 0, n \text{ is odd} \\ 1, n \text{ is even} \end{cases}$$

Hence, we can conclude that G_D (when duplicate a vertex v_k of degree 2) is a balanced DSD cordial graph when n is odd and unbalanced DSD cordial when n is even.

Example 3.3

Figure *1(a)* and *1(b)* illustrates the balanced DSD cordial graph $G(P_7)$ and unbalanced DSD cordial graph G_D obtained by duplicating a pendent vertex v_1 respectively.

Figure 2(a) and 2(b) illustrates the unbalanced DSD cordial graph $G(P_8)$ and unbalanced DSD cordial graph G_D obtained by duplicating a vertex v_1 respectively.

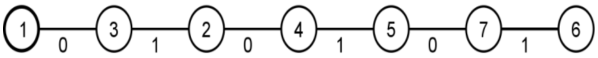

Figure 1(a) : Balanced DSD cordial graph G(P_7)

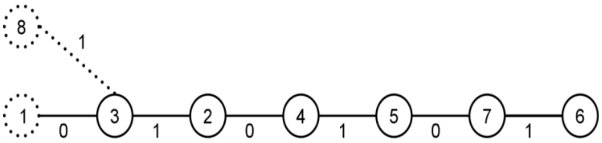

Figure 1(b) : Unbalanced DSD cordial graph G_D

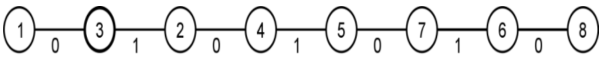

Figure 2(a) : Unbalanced DSD cordial graph G(P_8)

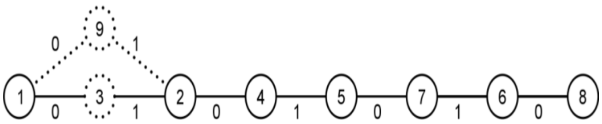

Figure 2(b) : Unbalanced DSD cordial graph G_D

Proposition 3.4

A graph obtained by duplication of a vertex v_k by a new vertex in a DSD cordial cycle C_n *(except $n \equiv 2 (mod\ 4)$)* is DSD cordial.

Proof

Let G be a cycle graph C_n. Let $v_1, v_2, \ldots, v_n$ are the vertices of the cycle C_n. In this graph, $|V(G)| = n$ and $|E(G)| = n$ By Proposition 2.12, we draw a DSD cordial cycle graph C_n. Now, without loss of generality we duplicate any arbitrary vertex v_k $(1 \leq k \leq n)$ in G by a vertex v_k' and construct a new graph G_D. In this graph G_D, $|V(G_D)| = n+1$ and $|E(G_D)| = n+2$. For DSD cordial labeling pattern, let the vertex labels are $\{1,2,\ldots,n+1\}$. Then, by labeling the graph G_D using definition 2.10 and the new vertex v_k' by $f(v_k') = n+1$, we get $|e_f(0) - e_f(1)| \leq 1$.

Hence G_D is also a divided square difference cordial graph.

Remark 3.5

From Proposition 3.4, in particular we get

$$|e_f(0) - e_f(1)| = \begin{cases} 0, n \text{ is even} \\ 1, n \text{ is odd} \end{cases}$$

Hence, we can conclude that G_D is a balanced DSD cordial graph when n is even and unbalanced DSD cordial graph when n is odd.

Example 3.6

Figure 3(a) and 3(b) illustrates the balanced DSD cordial graph $G(C_8)$ and balanced DSD cordial graph G_D obtained by duplicating a vertex v_3 respectively.

Figure 4(a) and 4(b) illustrates the unbalanced DSD cordial graph $G(C_9)$ and unbalanced DSD cordial graph G_D obtained by duplicating a vertex v_9 respectively.

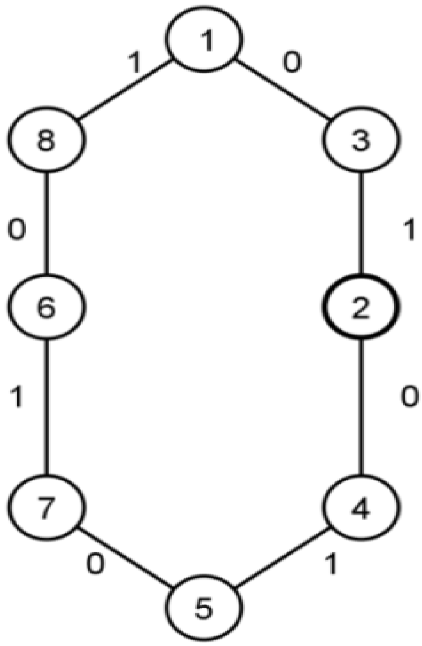

Figure 3(a) : Balanced DSD cordial graph G (C_8)

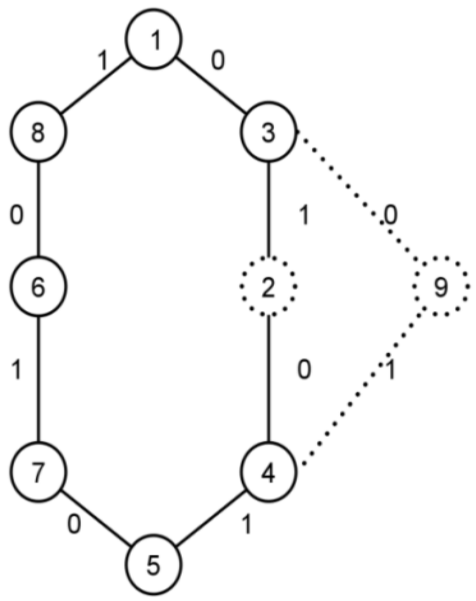

Figure 3(b): Balanced DSD cordial graph G_D

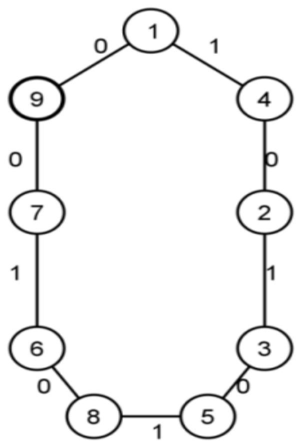

Figure 4(a) Unbalanced DSD cordial graph $G(C_9)$

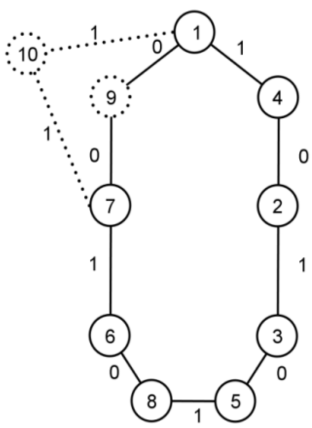

Figure 4(b) Unbalanced DSD cordial graph G_D

Proposition 3.7

A graph obtained by duplication of a vertex v_k $(1 \leq k \leq n)$ by a new vertex in a DSD cordial star graph $K_{1,n}$ is DSD cordial.

Proof

Let G be a star graph $K_{1,n}$. Let v be the central vertex and $v_1, v_2, \ldots, v_n$ are the end vertices of the star $K_{1,\,n}$. In this graph, $|V(G)|=n+1$ and $|E(G)|=n$. By Proposition 2.12, we draw a DSD cordial star graph $K_{1,n}$. Now, without loss of generality we duplicate any of the vertex v_k $(1 \leq k \leq n)$ in G by a vertex v_k' and construct a new graph G_D. In this graph G_D, $|V(G_D)|=n+2$ and $|E(G_D)|=n+1$. For DSD cordial labeling pattern, let the vertex labels are $\{1,2,\ldots,n+2\}$. Then, by labeling the graph G_D using definition 2.10 and the new vertex v_k' by $f(v_k')=n+2$, we get $|e_f(0)-e_f(1)|\leq 1$.

Hence G_D is also a divided square difference cordial graph.

Remark 3.8

From Proposition 3.7, in particular we get

$$|e_f(0)-e_f(1)| = \begin{cases} 0, n \text{ is odd} \\ 1, n \text{ is even} \end{cases}$$

Hence we can conclude that G_D is a balanced DSD cordial when n is odd and unbalanced DSD cordial when n is even.

Example 3.9

Figure 5(a) and 5(b) illustrates the unbalanced DSD cordial graph $G(K_{1,7})$ and balanced DSD cordial graph G_D obtained by duplicating a vertex v_4 respectively.

Figure 6(a) and 6(b) illustrates the balanced DSD cordial graph $G(K_{1,8})$ and unbalanced DSD cordial graph G_D obtained by duplicating a vertex v_1 respectively.

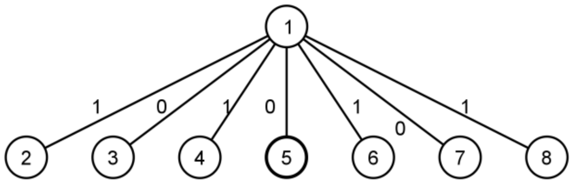

Figure 5(a) Unbalanced DSD cordial graph $G(K_{1,7})$)

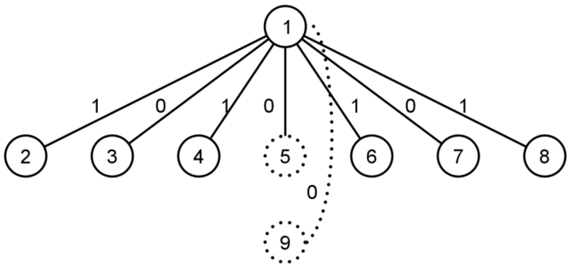

Figure 5(b) Balanced DSD cordial graph G_D

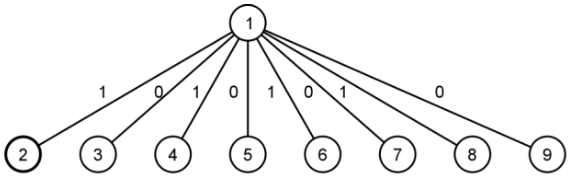

Figure 6(a) Balanced DSD cordial graph $G(K_{1,8})$

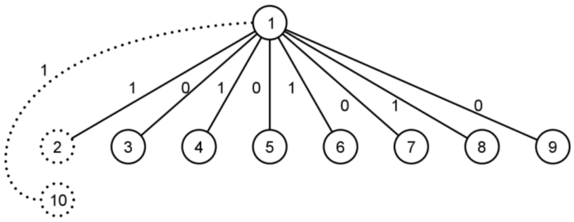

Figure 6(b) Unbalanced DSD cordial graph G_D

Proposition 3.10

A graph obtained by duplication of a vertex v_k $(1 \leq k \leq n)$ by a new vertex in a DSD cordial wheel graph W_n $(n \equiv 0,1 \pmod 4)$ is DSD cordial.

Proof

Let G be a DSD wheel graph W_n. Let u be the central vertex and $v_1, v_2, \ldots, v_n$ are the rim vertices of the wheel W_n. In this graph, $|V(G)|=n+1$ and $|E(G)|=2n$. By Proposition 2.13, we draw a DSD cordial wheel graph W_n. Now, without loss of generality we duplicate any of the vertex v_k $(1 \leq k \leq n)$ in G by a vertex v_k' and construct a new graph G_D. In this graph G_D, $|V(G_D)|=n+2$ and $|E(G_D)|=2n+3$.

For DSD cordial labeling pattern, let the vertex labels are $\{1,2,\ldots, n+2\}$. Then, by labeling the graph G_D using definition 2.10 and the new vertex v_k' by $f(v_k')=n+2$ we get $|e_f(0)-e_f(1)| \leq 1$.

Hence G_D is also a divided square difference cordial graph.

Remark 3.11

From Proposition 3.10, in particular we get $|e_f(0)-e_f(1)|=1$.

Hence we can conclude that G_D is a unbalanced DSD cordial graph.

Example 3.12

Figure 7(a) and 7(b) illustrates the balanced DSD cordial graph $G(W_8)$ and unbalanced DSD cordial graph G_D obtained by duplicating a vertex v_8 respectively.

Figure 8(a) and 8(b) illustrates the balanced DSD cordial graph $G(W_9)$ and unbalanced DSD cordial graph G_D obtained by duplicating a vertex v_3 respectively.

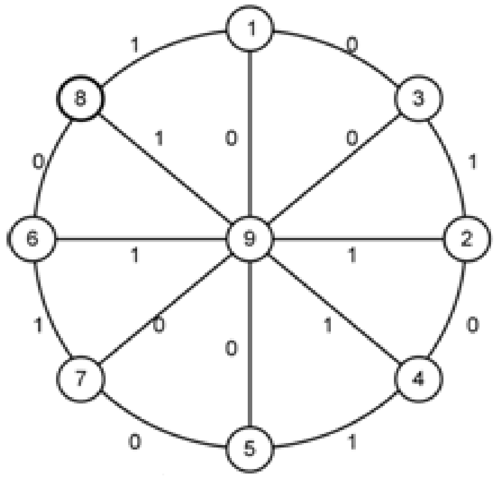

Figure 7(a) Balanced DSD cordial graph $G(W_8)$

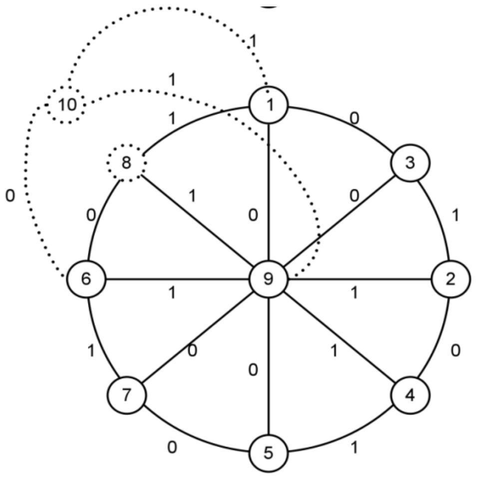

Figure 7(b) Unbalanced DSD cordial graph G_D

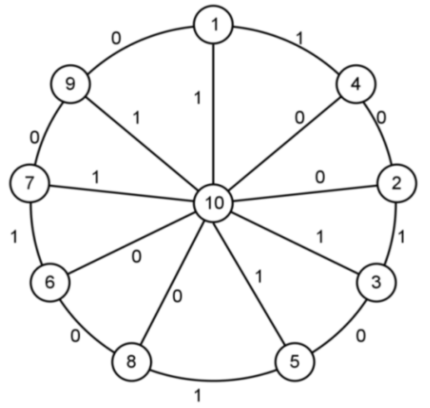

Figure 8(a) Balanced DSD cordial graph $G(W_9)$

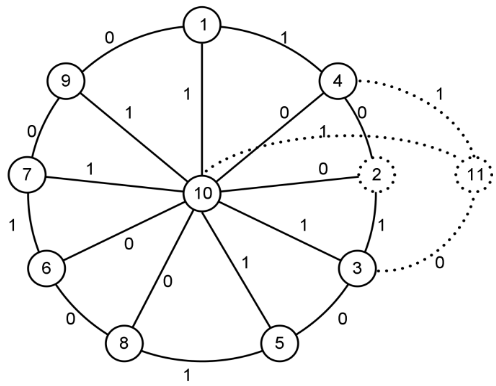

Figure 8(b) Unbalanced DSD cordial graph G_D

Proposition 3.13

A graph obtained by duplication of apex vertex v_k by a new vertex in a DSD cordial helm graph H_n $(n\equiv 0,1 \ (mod\ 4))$ is DSD cordial.

Proof

Let G be a helm graph H_n. Let x, $v_1,v_2,\ldots,v_n$, $u_1,u_2,\ldots,u_n$ are the vertices of H_n. Here x is the apex vertex, $v_1,v_2,\ldots,v_n$ are the vertices of the cycle C_n and $u_1,u_2,\ldots,u_n$ are the pendent vertices. In this graph, $|V(G)|=2n+1$ and $|E(G)|=3n$. By Proposition 2.13 we draw a DSD cordial helm graph H_n. Now, without loss of generality we duplicate apex vertex x in G by a vertex v_k' and construct a new graph G_D. In this graph G_D, $|V(G_D)|=2n+2$ and $|E(G_D)|=4n$. For DSD cordial labeling pattern, let the vertex labels are $\{1,2,\ldots,2n+2\}$. Then, by labeling the graph G_D using definition 2.10 and the new vertex v_k' by $f(v_k')=2n+2$ we get $|e_f(0)-e_f(1)|\leq 1$.

Hence G_D is also a divided square difference cordial graph.

Remark 3.14

From Proposition 3.13, in particular we get $0|e_f(0)-e_f(1)|=0$.

Hence we can conclude that G_D is a balanced DSD cordial graph when duplicating a apex vertex.

Example 3.15

Figure 9(a) and 9(b) illustrates the balanced DSD cordial graph $G(H_8)$ and balanced DSD cordial graph G_D obtained by duplicating apex vertex *x* respectively.

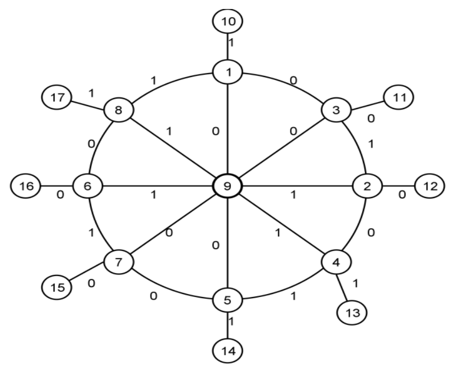

Figure 9(a) Balanced DSD cordial graph $G(H_8)$

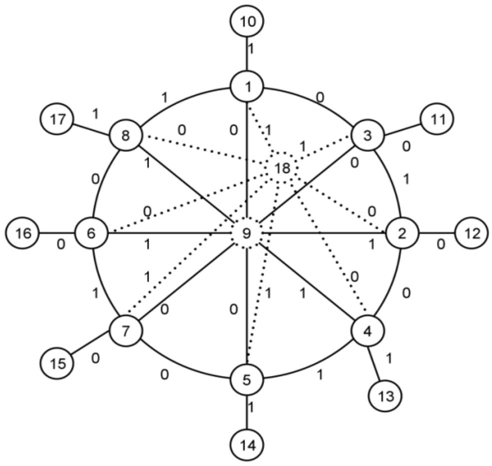

Figure 9(b) Balanced DSD cordial graph G_D

Proposition 3.16

A graph obtained by duplication of a vertex by a new vertex in a DSD cordial crown graph $C_n \odot K_1$ is divided square difference cordial.

Proof

Let G be a crown graph $C_n \odot K_1$. Let $u_1, u_2, \dots, u_n$ are the vertices of cycle C_n and $v_1, v_2, \dots, v_n$ are the vertices of n copies of K_1. In this graph, $|V(G)| = 2n = |E(G)|$. By Proposition 2.14 we draw a DSD cordial crown graph $C_n \odot K_1$.

Case i: duplicating a pendent vertex

Now, without loss of generality we duplicate a pendent vertex v_k $(1 \le k \le n)$ in G by a vertex v_k' and construct a new graph G_D. In this graph G_D, $|V(G_D)| = 2n+1$ and $|E(G_D)| = 2n+1$. For DSD cordial labeling pattern, let the vertex labels are $\{1,2,\dots,2n+1\}$. Then, by labeling the graph G_D using definition 2.10 and the new vertex v_k' by $f(v_k') = 2n+1$, we get $|e_f(0) - e_f(1)| \le 1$.

Case ii: duplicating a vertex u_k of degree three

Now, without loss of generality we duplicate a vertex u_k $(1 \le k \le n)$ of degree 3 in G by a vertex u_k' and construct a new graph G_D. In this graph G_D, $|V(G_D)| = 2n+1$ and $|E(G_D)| = 2n+3$. For DSD cordial labeling pattern, let the vertex labels are $\{1,2,\dots,2n+1\}$. Then, by labeling the graph G_D using definition 2.10 and the new vertex u_k' by $f(u_k') = 2n+1$, we get $|e_f(0) - e_f(1)| \le 1$.

Hence G_D is also a divided square difference cordial graph.

Remark 3.17

From Proposition 3.16, in particular we get $|e_f(0) - e_f(1)| = 1$.

Hence we can conclude that G_D is a unbalanced DSD cordial graph.

Example 3.18

Figure 10(a) and 10(b) illustrates the balanced DSD cordial graph $G(C_8 \odot K_1)$ and unbalanced DSD cordial graph G_D obtained by duplicating a vertex v_7 respectively.

Figure 11(a) and 11(b) illustrates the balanced DSD cordial graph $G(C_{10} \odot K_1)$ and unbalanced DSD cordial graph G_D obtained by duplicating a vertex u_9 respectively.

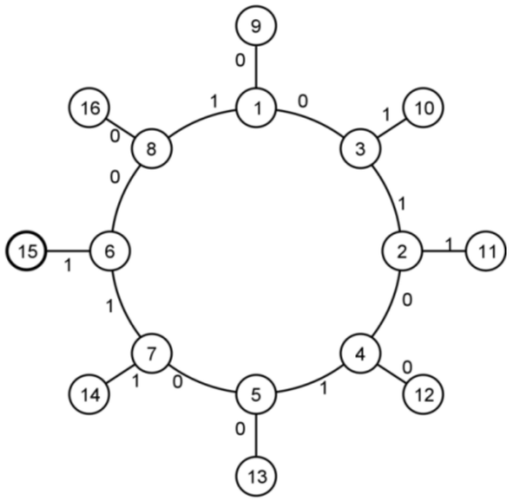

Figure 10 (a) Balanced DSD cordial graph $G(C_8 \odot K_1)$

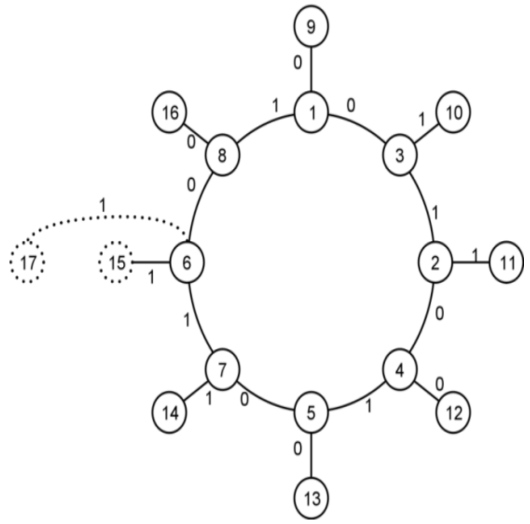

Figure 10 (b) Unbalanced DSD cordial graph G_D

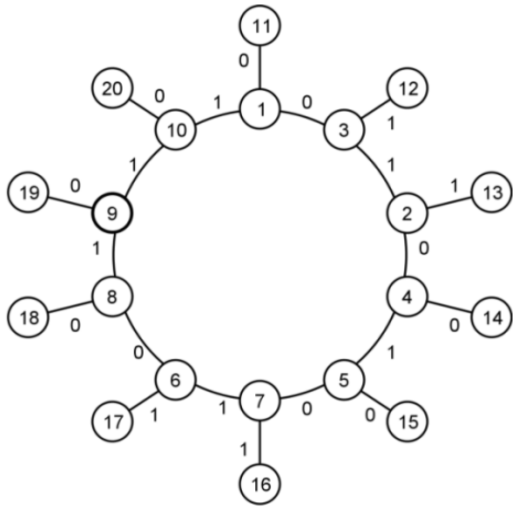

Figure 11 (a) Balanced DSD cordial graph $G(C_{10}\odot K_1)$

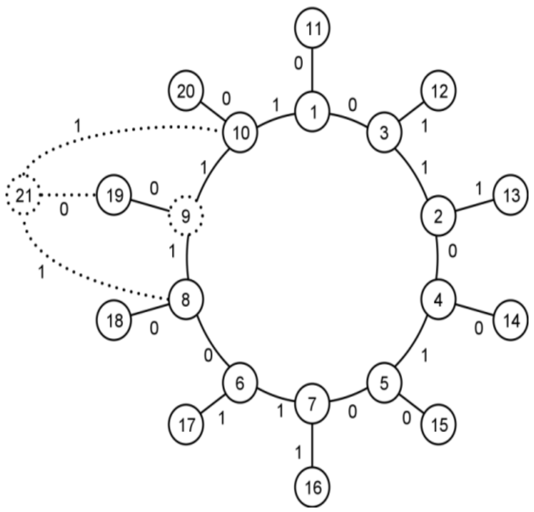

Figure 11 (b) Unbalanced DSD cordial graph G_D

Proposition 3.19

A graph obtained by duplication of a vertex by a new vertex in a DSD cordial comb graph $P_n\odot K_1$ is divided square difference cordial.

Proof

Let G be a comb graph $P_n\odot K_1$. Let $u_1,u_2,\ldots,u_n$ are the vertices of n copies of K_1 and $v_1,v_2,\ldots,v_n$ are the vertices of path P_n. In this graph, $|V(G)|=2n$ and $|E(G)|=2n-1$. By Proposition 2.14 we draw a DSD cordial comb graph $P_n\odot K_1$.

Case i: duplicating a vertex v_k $(k=1,n)$ of degree two

Subcase i: for $n\equiv 0,1,2 \pmod 4$

Now, without loss of generality we duplicate any of the vertex v_k $(k=n)$ in G by a vertex v_k' and construct a new graph G_D. In this graph G_D, $|V(G_D)|=2n+1$ and $|E(G_D)|=2n+1$. For DSD cordial labeling pattern, let the vertex labels are $\{1,2,\ldots,2n+1\}$. Then, by labeling the graph G_D using definition 2.10 and the new vertex v_k' by $f(v_k')=2n+1$, we get $|e_f(0)-e_f(1)|\le 1$.

Subcase ii: for $n\equiv 3 \pmod 4$

Now, without loss of generality we duplicate any of the vertex v_k $(k=1,n)$ in G by a vertex v_k' and construct a new graph G_D. In this graph G_D, $|V(G_D)|=2n+1$ and $|E(G_D)|=2n+1$. Then, by labeling the graph G_D using definition 2.10 and the new vertex v_k' by $f(v_k')=2n+1$, we get $|e_f(0)-e_f(1)|\le 1$.

Case ii: duplicating a vertex $v_k (1 < k < n)$ of degree three

Subcase i: for $n\equiv 0,1,2 \pmod 4$

Now, without loss of generality we duplicate any of the vertex v_k $(k=2,4,6\ldots)$ in G by a vertex v_k' and construct a new graph G_D. In this graph G_D, $|V(G_D)|=2n+1$ and $|E(G_D)$

$|=2n+2$. Then, by labeling the graph G_D using definition 2.10 and the new vertex v_k' by $f(v_k')=2n+1$, we get $|e_f(0)-e_f(1)|\leq 1$.

Subcase ii: for $n\equiv 3 \pmod 4$

Now, without loss of generality we duplicate any of the vertex v_k $(k=3,5,7\ldots)$ in G by a vertex v_k' and construct a new graph G_D. In this graph G_D, $|V(G_D)|=2n+1$ and $|E(G_D)|=2n+2$. Then, by labeling the graph G_D by definition 2.10 and the new vertex v_k' by $f(v_k')=2n+1$, we get $|e_f(0)-e_f(1)|\leq 1$.

Hence G_D is also divided square difference cordial graph.

Remark 3.20

From Proposition 3.19, in particular we get $|e_f(0)-e_f(1)|=1$ when duplicating a vertex of degree two and $|e_f(0)-e_f(1)|=0$ when duplicating a vertex of degree three.

Hence, we can conclude that G_D is a unbalanced or balanced DSD cordial graph when duplicate a vertex of degree two or degree three respectively.

Example 3.21

Figure 12(a) and 12(b) illustrates the unbalanced DSD cordial graph $G(P_8 \odot K_1)$ and balanced DSD cordial graph G_D obtained by duplicating a vertex v_4 respectively.

Figure 13(a) and 13(b) illustrates the unbalanced DSD cordial graph $G(P_7 \odot K_1)$ and unbalanced DSD cordial graph G_D obtained by duplicating a vertex v_1 respectively.

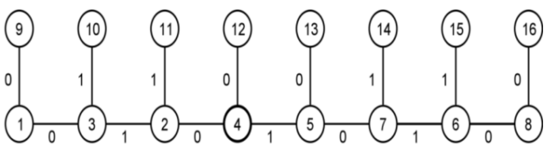

Figure 12(a)Unbalanced DSD cordial graph G ($P_8 \odot K_1$)

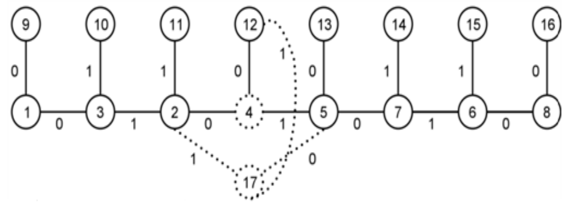

Figure 12 (b) Balanced DSD cordial graph G_D

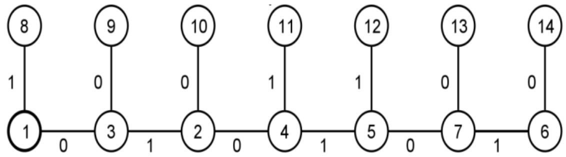

Figure 13(a) Unbalanced DSD cordial graph G ($P_7 \odot K_1$)

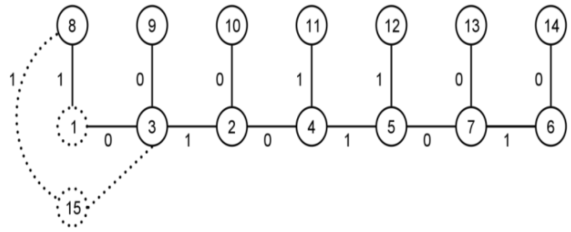

Figure 13(b) Unbalanced DSD cordial graph G_D

IV. CONCLUSION

A In this article, we have discussed the divided square difference (DSD) cordial labeling in the context of duplication of a vertex by a new vertex in DSD cordial graphs such as path graph, cycle graph, star graph, wheel graph, helm graph, bistar graph, crown graph and comb graph. To investigate DSD labeling on the graphs obtained by duplication of a vertex by a new vertex for other graph families is an open area of research.

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