



Elastic Damping Gravitational Effect of Two Different Sized Particle – Particle Contact Using Discrete Element Method

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ABSTRACT

The objective of this contribution is to exist a numerical simulation technique to model the collision of particles in a plane using object-oriented procedures. This approach is based on the second law of Newton for the translational motion of each particle in the granular material. This comprises a possession track of all forces acting on each particle at every time-step. The back-ground version of DEM and time integration algorithm are established and executed into C++ code. A simple test is regarding the application of time- integration algorithm by particle-particle interaction for which analytical expression exist. In this paper elastic damping gravitational effect due to particle – particle contact on different sized particles are investigated.

Keywords:—DEM simulation, Granular materials, Elastic effect, damping effect;

I. INTRODUCTION

A granular material is a conglomeration of many discrete solid particles which is characterized by a loss of energy due to dissipative collisions whenever the particles interact. They can be considered as the fourth state of matter which is very different from solids, liquids or gas. Granular materials are very simple. If the particles are non-cohesive, then the forces

between them are basically only repulsive so that the shape of the material is determined by external boundaries and gravity. Practically many solid particles which we make use of in the kitchen are granular particles like sugar, rice, coffee, cereals etc. Walking outside we step on the soil which is again a particulate and hence falls under the category of granular matter. The unusual and unique character displayed by granular material systems have led to a resurgence of interest within several scientific and engineering disciplines ranging from physics, soil mechanics and chemical engineering (Jaeger and Nagel [1992]; Behringer [1995]; Bideau and Hansen [1993]; Jaeger et al. [1994, 1996a]). Much of the engineering literature has been dedicated to understanding how to deal with these materials. Prominent contributions in the literature include Coulomb [1773], who proposed the ideas of static friction; Faraday [1831], who discovered the convective instability in a vibrated container filled with powder, and Reynolds [1885], who introduced the motion of dilatancy, which implies that a compacted granular material must expand in order for it to undergo shear. Processes involving particulate or granular flows are prevalent throughout the pharmaceutical, chemical, energy, food handling, mineral processing, powder metallurgy, and mining industries.

In addition, numerous phenomena found in nature involve such material flows.

The discrete element method (DEM), originally developed by Cundall and Strack [1971, 1979], has been used successfully to simulate chute flow (Dippel et.al 1996), heap formation (Luding, 1997), hopper discharge (Thompson and Grest, 1991; Ristow and Herrmann, 1994), blender segregation (Wightman et al., 1998; Shinbrot et al., 1999; Moakher et al., 2000) and flows in rotating drums (Ristow, 1996; Wightman et al., 1998). The DEM allows for the simulation of particle motion and interaction between the particles. DEM considering not only the obvious geometric and material effects such as particle shape, material non-linearity, viscosity, friction, etc., but also the effect of various physical fields of surrounding media, level of chemical reactions (Kantor et al. 2000). One of the most auspicious area of future applications of discrete element method seems to be geotechnical engineering. The discrete approach assumes the soil is an assembly of granular or discrete particle.

II. DISCRETE STATE FORMULATION

The dynamic behavior of granular media is considered as the dynamics of each particles. The overall response of media is foretold by the behaviour of individual particles and the dynamics of particles is evaluated by applying the second Newton's law. Detection of interaction force between contacting particles is considered by discrete approach is one of the most important issues. The interaction forces of each contacting particle are locally resolved based on actual geometry of kinematic contact between two spherical particles, inter-particle contact forces and boundary conditions.

Granular material is considered as a collection of frictional elastic spherical

particles and it is termed as discrete elements. The particles are assumed to be composed of spherical particles with same radii R_i . The granular particles are assumed to be deformable bodies, deforming each other by normal and shear force. The composition of media is time-dependent because distinct particle changes their position by free rigid body motion or by contacting with neighbor particles or walls. Each particle may be in contact with other particles. The boundary conditions of media are determined by planes and treated as particles with an infinite radius and mass

III. GEOMETRY OF KINEMATIC CONTACT OF SPHERICAL PARTICLES

Consider two particles i and j be in contact with position vectors x_i and x_j with center of gravity lying at O_i and O_j having linear velocities v_i and v_j .

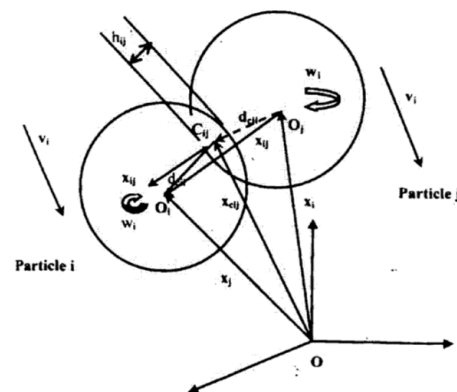


Figure 1. Inter Particle Contact Between two Identical Spheres i and j

The contact point C_{ij} is defined to be at the center of the overlap area position vector x_{cij} . The vector x_{ij} of the x_{ij} relative position point from the center to gravity of particle i to that of particle j is defined as $x_{ij} = x_i - x_j$. The depth of overlap is h_{ij} . Unit vector in the normal direction of the contact surface through the center of the overlap area is denoted by n_{ij} . It extends from the contact point to the inside of the particle i as $n_{ij} = -n_{ji}$. Since the particle shape is assumed to be spherical, for sphere of any

dimension the contact parameters can be written as follows:

$$h_{ij} = \begin{cases} R_i + R_j - |x_{ij}|, & |x_{ij}| < R_i + R_j \\ 0, & |x_{ij}| \geq R_i + R_j \end{cases}$$

Where R_i is the radius of the particle. The relative velocity of the contact point is defined as

$$v_{ij} = v_{cij} - v_{cji}$$

4. INTER PARTICLE CONTACT FORCE

The contact force between particles can be expressed as the sum of normal and tangential components;

$$F_{ij} = F_{n,ij} + F_{t,ij}$$

The contact forces between particles depend on the overlap geometry, the properties of the material and the relative velocity between the particles in the contact area and it is modeled as spring, dash-pots and a friction slider. If the particles are in perfect contact model, it is required to describe the effects of elasticity, energy loss through internal friction and attraction on the contact surface for the contact force calculations. The normal component of contact force between particles can be expressed as the sum of elastic force and viscous force.

$$F_{n,ij} = F_{n,ij,elastic} + F_{n,ij,viscous}$$

Normal elastic force is based on the linear Hooke's law of a spring with a spring stiffness constant k_{nij} and is given by the expression,

$$F_{n,ij,elastic} = K_{n,ij} h_{ij} n_{ij}$$

Normal viscous force is dissipated during real collisions between particles. The linear dependency of force on the relative velocity of the particles at the contact point with a

constant normal dissipation coefficient γ_n and is expressed as

$$F_{n,ij,viscous} = -\gamma_n m_{ij} v_{n,ij}$$

Force acting on i^{th} particle F_i is,

$$F_i = m_i g + \sum_{\substack{j=i \\ j \neq i}}^N F_{n,ij} + \sum_{\substack{j=i \\ j \neq i}}^N F_{t,ij}$$

i.e., sum of gravitational force and contact force

V. COMPUTER IMPLEMENTATION

The key computational tasks of DEM at each time step of contact particle can be summarized as follows:

- Finding of contacts between a particle i and j .
- Calculation of contact forces from relative displacement between particles
- Summary of contact forces to determine the total unbalanced force
- Computation of acceleration from force
- Velocity and displacement by integrating the acceleration
- Updating the position of particles

VI. RESULT AND DISCUSSION

6.1. Elastic, Damping and Gravitational Impact of different sized Particles by Particle – Particle contact using Discrete Element Method

This test simulates the different sized particles collide with each other, and thus particle-particle contact force are formed. Here test confirms the particle-particle interaction of different sized particles with normal stiffness parameter is 3×10^5 units. In this test the gravitational effect causes a

change in force, displacement and kinetic energy of a particle. Tangential force is set to be zero.

6.1.1. Effect of Elastic, Damping and Gravitational Force with respect to displacement

6.1.1.1. Particles with difference in mass and radius

Considers two different sized particles with different radius and mass. The first particle moving with the initial velocity and collide the second particle, then the elastic damping gravitational force reaches a peak during contact. Elastic damping gravitational force with respect to displacement on large particle contact with small particle is shown in figure 2 and figure 3 shows the results for an elastic damping gravitational force on smaller particle contact with the larger particle. Elastic damping gravitational force is in the x-direction and displacement in the y-directions. In the two cases particles attain same effect of elastic damping gravitational force with respect to displacement that is change in radius and mass does not change the elastic damping gravitational force on particles with different displacement. Larger particle collide with smaller particle attain more displacement than smaller particle collide with larger particle.

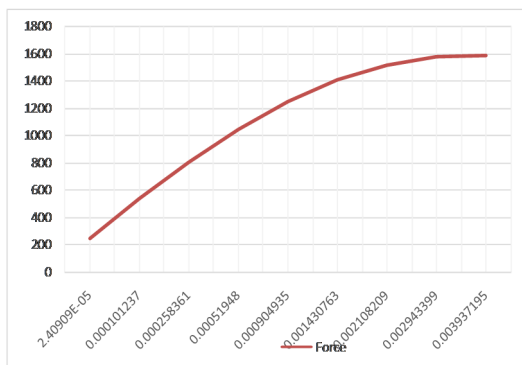


Figure 2 Elastic damping and gravitational force on larger particle collide with smaller particle with change in Displacement

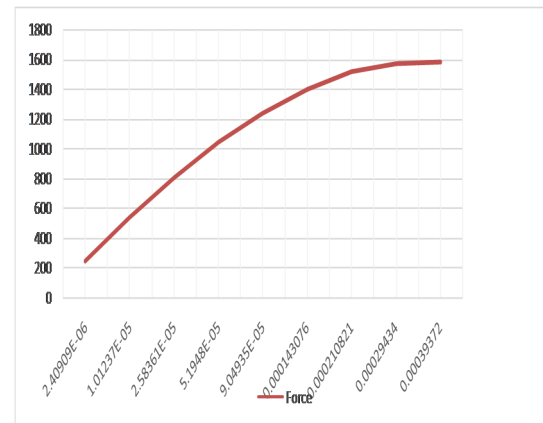


Figure 3: Elastic damping and gravitational force on larger particle collide with smaller particle with change in Displacement

6.1.1.2. Particles with same radius and different mass.

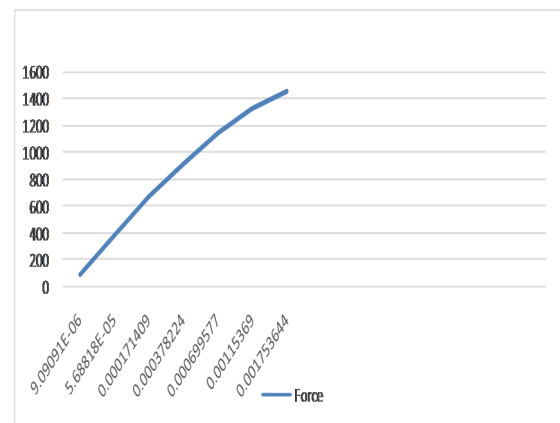


Figure 4: Elastic damping gravitational effect of particles with larger mass collides with smaller mass particles with change in displacement

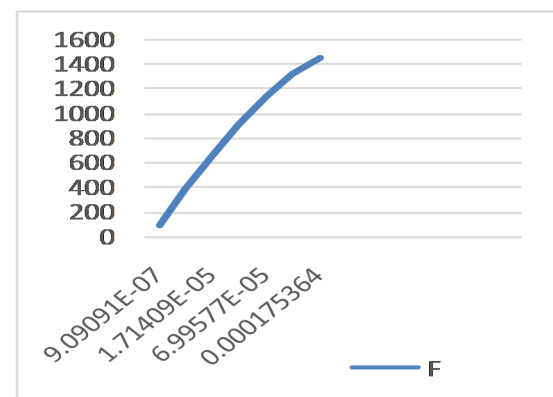


Figure 5: Elastic damping gravitational effect of particles with smaller mass collides with larger mass particles with change in displacement

Effect of elastic damping gravitational force with respect to displacement on particles with larger mass collide with smaller mass with same radius as shown in figure 4. Effect of elastic damping gravitational force with respect to displacement on particles with smaller mass collide with larger mass with same radius as shown in figure 5. When particle with larger mass collide with smaller mass and particle with smaller mass collide with larger mass in both case we get same elastic damping gravitational force with respect to different displacement. Here particle with larger mass collide with smaller mass attain maximum displacement than other.

6.1.1.3. Particles with same mass but different radius.

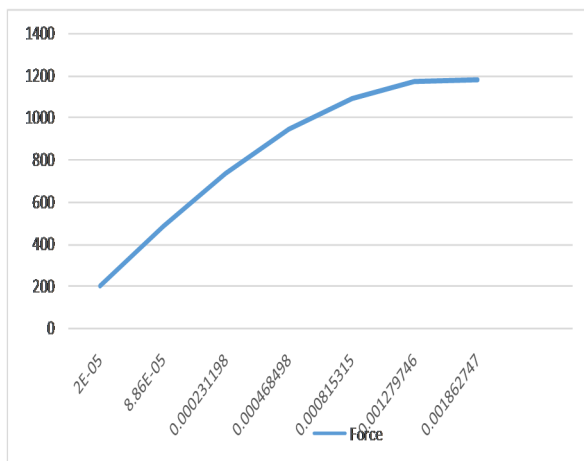


Figure 6: Elastic damping gravitational effect of particles with larger radius collides with smaller radius particles with change in displacement

Effect of elastic damping gravitational force with respect to displacement on particles with larger radius collide with smaller radius as shown in figure 6 and elastic damping gravitational force with respect to displacement on smaller particle collide with larger particle as shown in figure 7. From the result we can say in both cases we get same elastic damping gravitational force with respect to displacement. Displacement and elastic damping gravitational force

obtained by two particles are same. Particles with same radius but different mass and same mass but different radius in these two cases elastic damping gravitational forces are different with different displacement

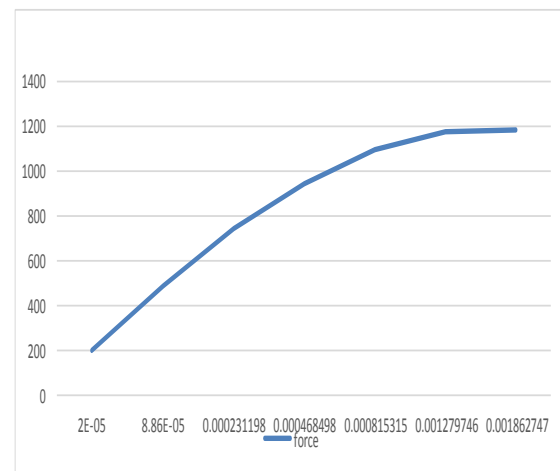


Figure 7. Elastic damping gravitational effect of particles with smaller radius collides with larger radius particles with change in displacement

- By comparing the three cases.
- Least elastic damping gravitational force attain by particles with same mass but different radius.
- Maximum elastic damping gravitational force is obtained by particles with difference in mass and radius

6.1.2. Elastic, Damping and gravitational Effect of Force with respect to Time

Considers two different sized particles with different radius and mass. The incoming velocity magnitude of first particle was set at 1 m/s and the initial position is 5m.

6.1.2.1. Particles with difference in mass and radius

Considers two different sized particles with different in radius and mass. Larger particle collide with smaller particle as shown in Figure 8. If the particles have different radius and mass but it attain same elastic

damping gravitational force with respect to time. But collision time and duration of collision of particles with difference in mass and radius are also different. When we consider the first particle as smaller and the second as larger is shown in figure 9. In these two cases particles attain same effect of elastic damping gravitational force with respect to time that is change in radius and mass does not change the elastic damping gravitational force on a particle with respect to time.

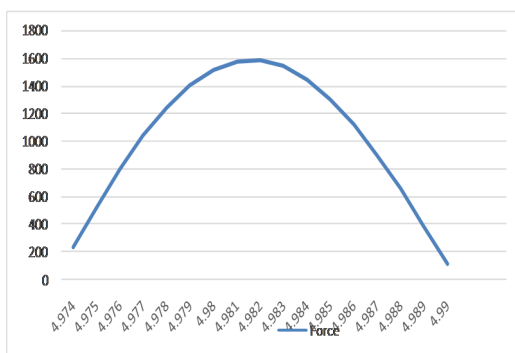


Figure 8. Elastic damping gravitational effect of larger particles collides with smaller particles with change in Time

From the result we can say that elastic damping gravitational force with respect to change in time has no effect on collision with larger particle collide the smaller particle or smaller particle collide the larger particle in both case we get the same elastic damping gravitational force. Elastic damping gravitational force with respect to time step in both case are same.

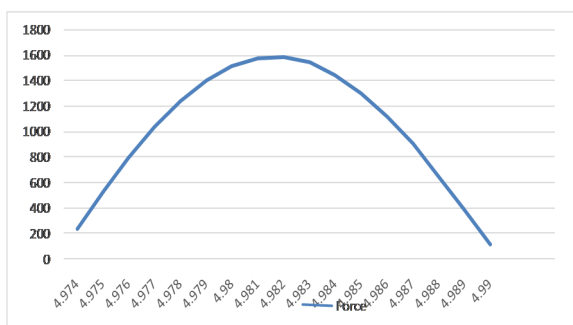


Figure 9. Elastic damping gravitational effect of smaller particles collides with larger particles with change in Time

6.1.2.2. Particle with same radius but different mass

From the result we can say that elastic, damping and gravitational force with respect to change in time on particles with same radius and different mass, by considering first case as particle with larger mass collide with smaller mass and second case with smaller mass particle collide with larger mass particle in both case we get same elastic damping gravitational force. Thus change in time does not change the elastic, damping and gravitational force on a particle.

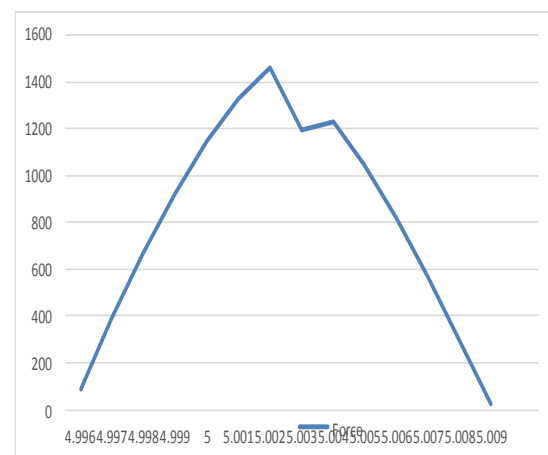


Figure 10: Elastic damping gravitational effect of particles with larger mass collides with smaller mass particles with change in Time

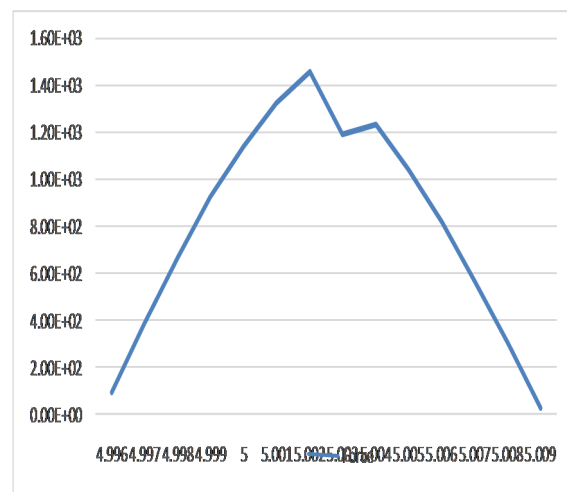


Figure 11. Elastic damping gravitational effect of particles with smaller mass collides with larger mass particles with change in Time

6.1.2.3. Particle with same mass but different radius

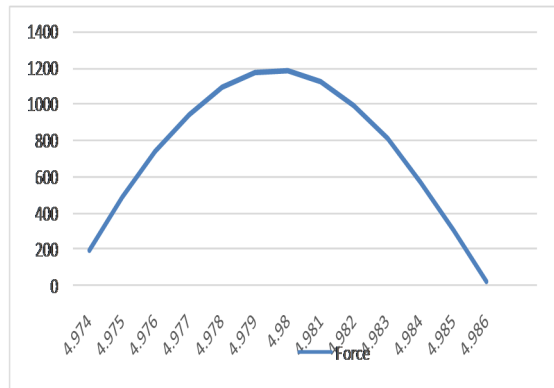


Figure 12. Elastic damping gravitational effect of particles with larger radius collides with smaller radius particles with change in Time

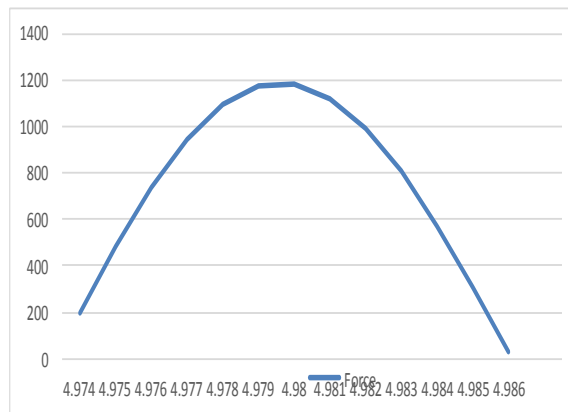


Figure 13. Elastic damping gravitational effect of particles with smaller radius collides with larger radius particles with change in Time

Particles with same mass but different radius are calculated. Larger radius particle collide with smaller radius are shown in figure 12 and figure 13 shows the particle with smaller radius collide with larger radius particle.

By comparing the three cases:

- Elastic damping gravitational effect different sized particles and particles with same mass but different radius collides earlier than particle with same radius but different mass.
- Elastic damping gravitational effect duration of collision for particles with

same mass but different radius and same radius but different mass are same.

- Elastic damping gravitational effect particles with same radius but different mass collide later. But the duration of collision for same mass but different radius and same radius but different mass are same.

6.1.3. Kinetic Effect of different sized particles with respect to Time

Considers the elastic damping gravitational impact of two different sized particles with velocity and position are not same. The first particle is moving with the same velocity and collide with the second particle, then the velocity of two particles changes and first particle moves in the direction of the second particle.

6.1.3.1. Particles with different mass and radius

The first particle having larger radius and mass moving with initial velocity and collide the second particle, the kinetic energy varies and reduces to a constant level as shown in figure 14. The first particle occurs a small decreases in velocity results in the gradual decrease in Kinetic energy of the first particle. If the velocity is constant then Kinetic energy will also be constant. If first particle continuously increases in velocity, the Kinetic energy too continuously varies. When there is no collision the velocity remains constant. Particles with smaller radius and mass collide with larger radius and mass are shown in figure 15.

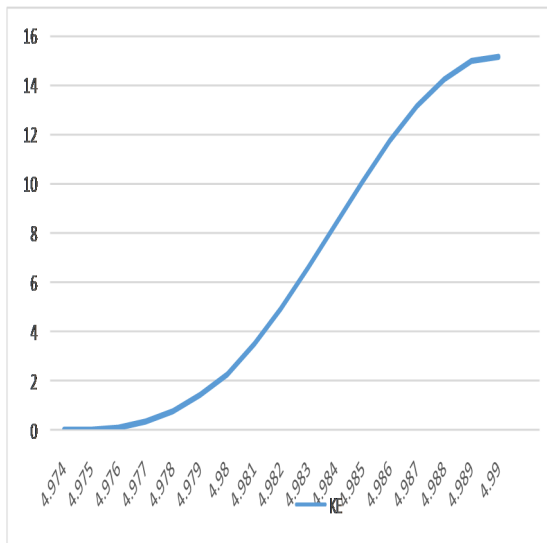


Figure 14. Elastic damping gravitational effect of larger particles collides the smaller particles with change in kinetic energy with respect to Time

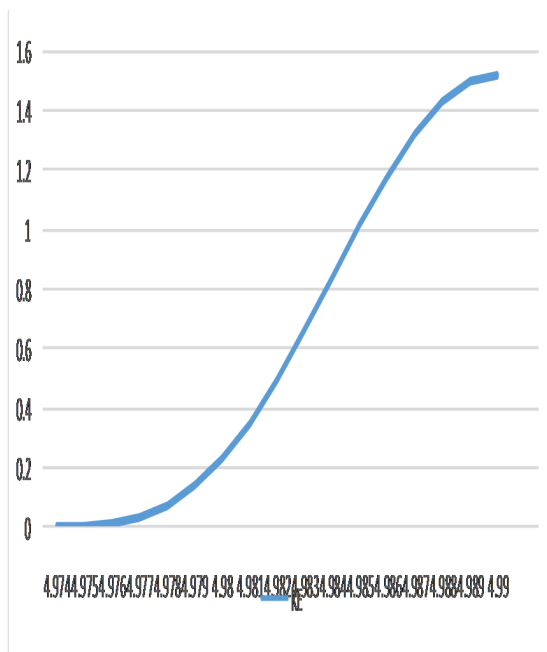


Figure 15. Elastic damping gravitational effect of smaller particles collides the larger particles with change in kinetic energy with respect to Time

Consider larger particle collide the smaller particle and smaller particle collide with larger particle in both case there in an increase in kinetic energy occurred. But larger particle collide with smaller attain more kinetic energy than particle with smaller particle collide with larger particle.

6.1.3.2. Particles with same radius but different in mass

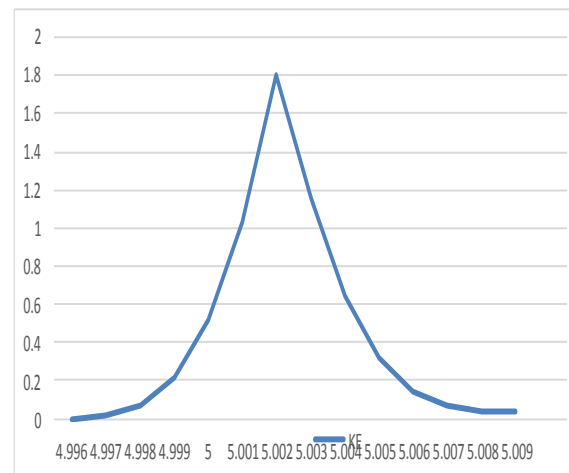


Figure 16. Elastic damping gravitational effect of particles with larger mass collides the particles with smaller mass with change in kinetic energy with respect to Time

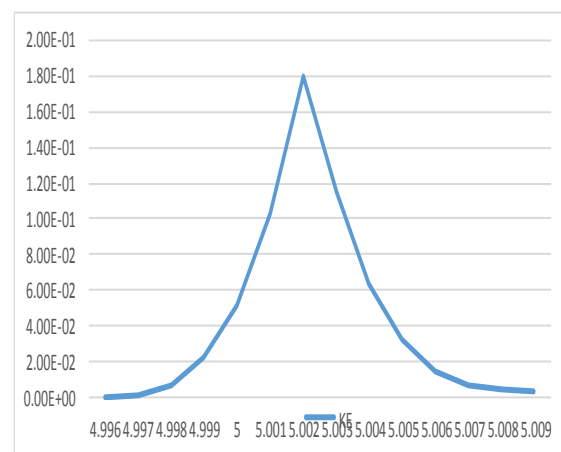


Figure 17. Elastic damping gravitational effect of particles with larger mass collides the particles with smaller mass with change in kinetic energy with respect to Time

First particle with larger in mass and the second particle with smaller in mass as shown in Figures 16 and first particle with smaller in mass and the second particle with larger in mass as shown in figure 17. Larger mass collide with smaller mass particle attain more kinetic energy than smaller mass particle collide with larger mass particle. But the time of collision for two cases are same. In this case kinetic energy of the particle increases to a limit and then

kinetic energy decreases and reaches a constant.

6.1.3.3. Particles with same mass but different in radius

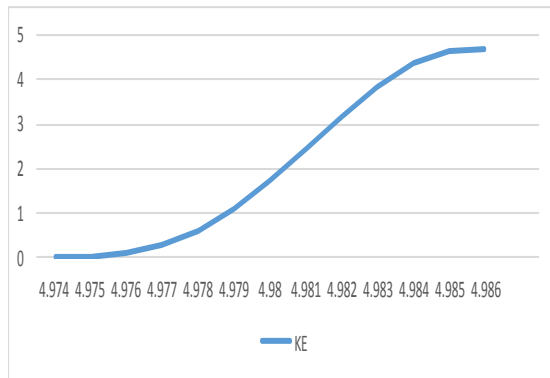


Figure 18. Elastic damping gravitational effect of particles with larger radius collides the particles with smaller radius with change in kinetic energy with respect to Time

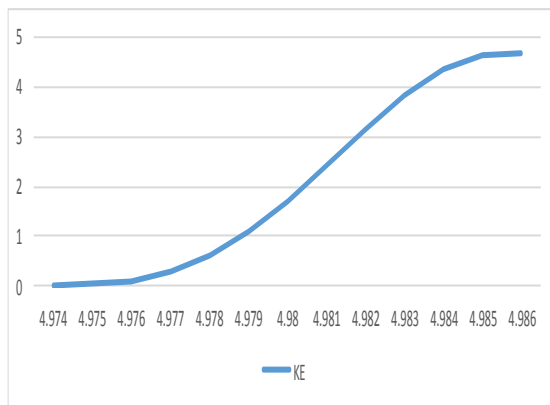


Figure 19. Elastic normal effect of second particle with same mass and differ in radius

If we consider particles with different radius and same mass. First particle with larger radius and the second particle with smaller in radius as shown in Figures 18 and first particle with smaller in radius and the second particle with larger in radius as shown in Figures 19. In the two cases the velocity obtained is same so kinetic energy acquired by the particle is also same. Therefore kinetic energy with respect to time on both particles are same.

By comparing the three cases:

- Elastic damping gravitational effect of larger particle collide with smaller particle attain more kinetic energy.
- Elastic damping gravitational effect of smaller mass particle collide with larger mass particle at a constant radius acquire least kinetic energy.
- Elastic damping gravitational effect of particles with larger radius collide with smaller radius and smaller radius particle collide with larger mass at constant mass acquire same kinetic energy.
- Elastic damping gravitational effect of particle with different mass but same radius acquire an increase in kinetic energy and then decreases to a constant.

6.1.4. Displacement of different sized particles with respect to Time

When two different sized particles collide then their displacement with respect to time also changed.

6.1.4.1. Particles with different mass and radius

When two different sized particles collide then the effect of displacement with respect to time are calculated.

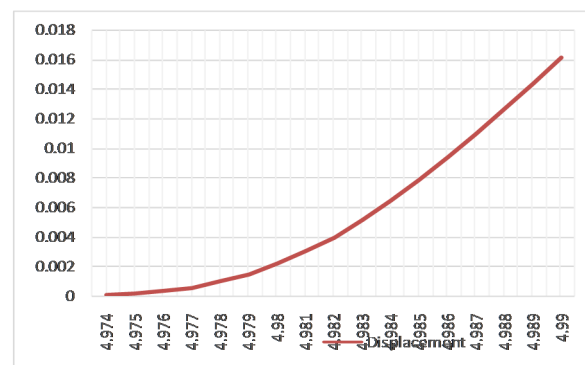


Figure 20. Elastic damping gravitational effect of larger particles collides the smaller particles with change in displacement with respect to Time

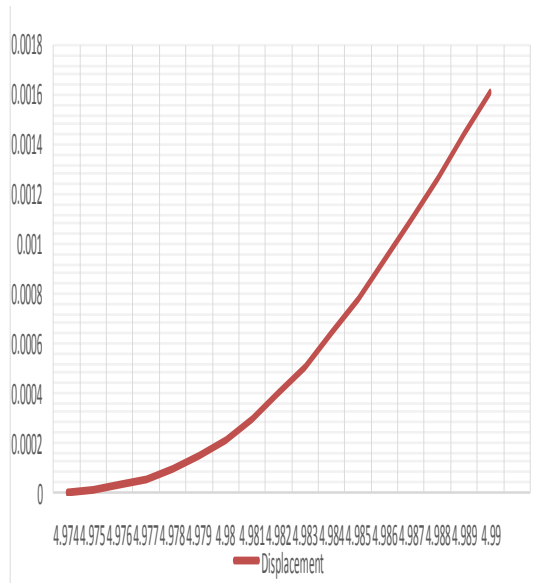


Figure 21. Elastic damping gravitational effect of smaller particles collides the larger particles with change in displacement with respect to Time

Figure 20 shows the larger particle collide with smaller particle and figure 21 shows the smaller particle collide with larger particle. In both case we can see that more displacement is obtained in larger particle collide with smaller particle.

6.1.4.2. Particles with same radius and different in mass

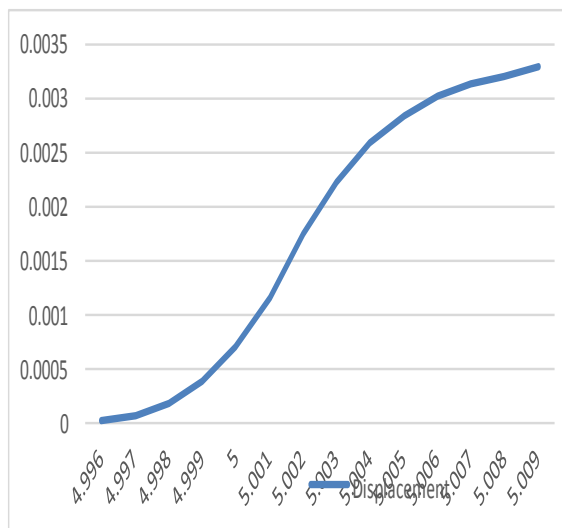


Figure 22. Elastic damping gravitational effect of particles with larger mass collides the particles with smaller mass by change in displacement with respect to Time

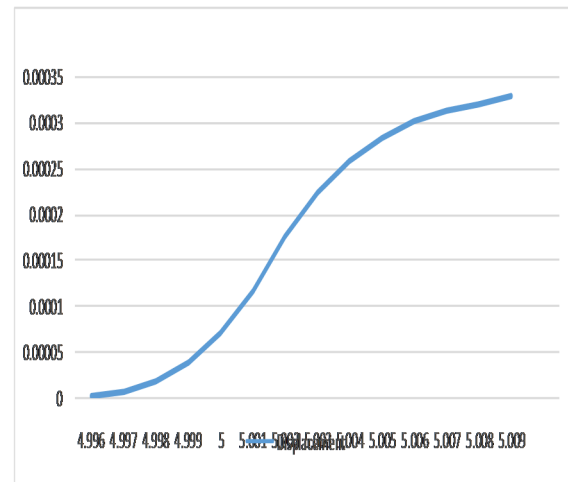


Figure 23. Elastic damping gravitational effect of particles with smaller mass collides the particles with larger mass by change in displacement with respect to Time

Figure 22 shows the particle with larger mass collide particle with smaller mass and figure 23 shows the particle with smaller mass collide particle with larger mass. In both case we can see that more displacement is obtained in particle with larger mass collide particle with smaller mass.

6.1.4.3. Particles with same mass and different in radius

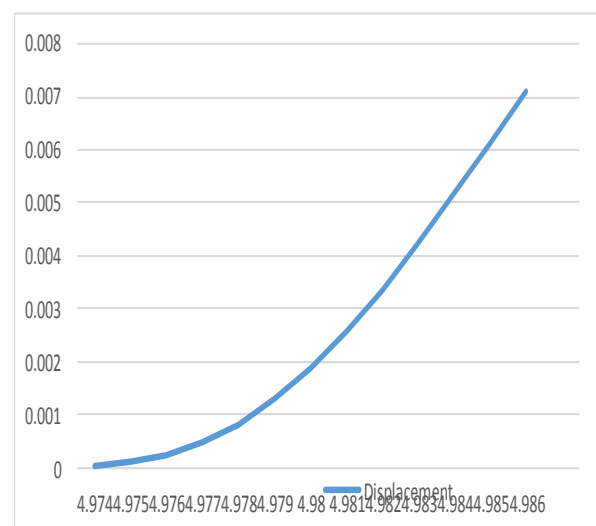


Figure 24. Elastic damping gravitational effect of particles with larger radius collides the particles with smaller radius by change in displacement with respect to Time

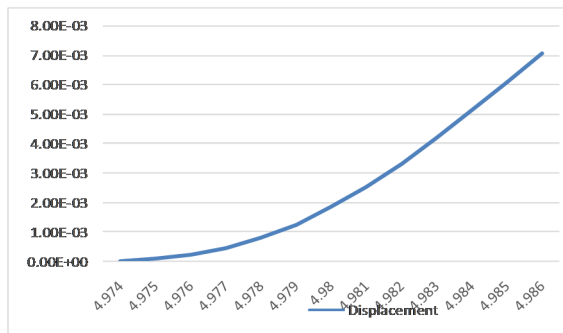


Figure 25. Elastic damping gravitational effect of particles with smaller radius collides the particles with larger radius by change in displacement with respect to Time

Figure 24 shows larger particle collide with smaller particle and figure 25 shows the smaller particle collide with larger particle. In both case we get same displacement with respect to time.

By comparing the three cases:

- Elastic damping gravitational effect of larger particle collide with smaller particle acquire more displacement.
- Elastic damping gravitational effect particle with small mass collide with large mass at same radius attain least displacement compared to other.
- Elastic damping gravitational effect particle with larger radius collide with smaller radius and smaller radius particle collide with larger size particle attain same displacement with respect to time.

VII. CONCLUSION

The result obtained in the present investigation may be generally described as follows:

- The described discrete element model composed of visco-elastic spherical particles is implemented into the developed C++ code.
- The analytical solutions for the impact of two different sized spheres

have been examined and derived. The normal force reaches a peak during contact.

- Elastic damping gravitational force with respect to different sized particle with respect to time and displacement are analyzed.
- Elastic damping gravitational effect on kinetic energy on a particle with different time step are analyzed.
- Elastic damping gravitational effect on displacement on a particle with different time step are analyzed.
- Elastic damping gravitational effect of different sized particle with mass change and radius change are analyzed.

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