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Effects of the Thermal Radiation on the Boundary Layer Flow over an Exponentially Stretching Sheet in the Presence of Viscous Dissipation

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ABSTRACT

A systematic study has been performed on the two dimensional boundary layer flow of an incompressible fluid over an exponentially stretching sheet with the thermal radiation and viscous dissipation is taken into account. The non-dimensional governing equations are transformed into ordinary differential equations with the help of similarity transformations. The reduced nonlinear ordinary differentials are solved numerically by utilizing the MATLAB in built solver bvp5c. The numerical results for the skin friction, Nusselt number, the velocity and temperature distributions are computed, examined and discussed.

Keywords:—*Thermal radiation, Stretching Sheet, Viscous dissipation, Skin friction, Nusselt number.*

I. INTRODUCTION

The investigation of flow over a stretching sheet has created much enthusiasm for late years in perspective on its various modern applications, for example, the streamlined extrusion of plastic sheets, the boundary layer along a liquid film, condensation procedure of metallic plate in a cooling bath and glass, and furthermore in polymer industries and which considered the stretching flow problems, different aspects of the problems have been examined by many authors, for example, Gupta and Gupta[1] investigated on heat and mass transfer on a stretching sheet with suction and blowing.

Sajid and Hayat [2] described the influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet. Chemically reactive solute distribution in MHD boundary layer flow over a permeable stretching sheet with suction or blowing was investigated by Bhattacharyya and Layek [3]. MHD boundary layer flow due to an exponentially stretching sheet with radiation effect was presented by Anuar Ishak[4]. Numerical solution of the boundary layer flow over an exponentially stretching sheet with thermal radiation was given by Biliana and Nazar [5]. Patel and Timol [6]studied and analyzed the numerical solution of steady two-dimensional MHD forward stagnation point flow. Swain et.al [7] analyzed the flow over exponentially stretching sheet through porous medium with heat source/ sink**.** Tripathy et.al [8] considered and examined the chemical reaction effect on MHD free convective surface over a moving vertical plane through porous medium. Goud[9] presented the MHD flow past a vertical oscillating plate with radiation and chemical reaction in porous medium-finite difference method. Azhar et.al [10] considered and examined analytic Solution for Fluid Flow over an Exponentially Stretching Porous Sheet with Surface Heat Flux in Porous Medium by Means of Homotopy Analysis Method.

From the above studies, it is noticed that the two dimensional boundary layer flow over an exponentially stretching sheet with thermal radiation and viscous dissipation. Using the similarity transformations, the governing equations are reduced into a set of nonlinear ordinary differential equations which are then solved numerically using the MATLAB in built solver bvp5c. The computed results are plotted in figures and discussed physically in different circumstances.

II. Mathematical Formulation

In this analysis, consider a steady, two-dimensional boundary layer flow of an incompressible viscous fluid over a stretching sheet in the presence of radiation effects. Where the sheet is along the y axis and in which the x-axis is taken along the stretching sheet in the direction of the fluid flow. It is also assumed that the viscous dissipation terms are taken into account. Under the usual boundary layer approximations the governing equations are as follows:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2} \quad (2)$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}+\frac{\mu}{\rho C_p}\left(\frac{\partial T}{\partial y}\right)^2 \quad (3)$$

The proper boundary condition are given by

$$\begin{aligned}&u=U_0e^{x/L}, v=0, T=T_\infty+T_0e^{x/2L} \quad at\ y\to 0\\&u\to 0,\quad T\to T_\infty, \qquad as\ y\to\infty\end{aligned} \quad (4)$$

Where U_0, T_0 and L is the stretching velocity, temperature at the plate and characteristic length of the plate Temperature far away from the plate is T_∞. Where q_r is the Rosseland approximation of the radiation effect for an optically thick layer given by

$$q_r=-\frac{4}{3}\frac{\sigma^*}{k^*}\frac{\partial T^4}{\partial y} \quad (5)$$

If the temperature variance within the flow are appropriately small, by expanding in T^4 to the Taylors series about T_∞ which after ignoring higher-order terms takes form

$$T^4\cong 4T_\infty^3T-3T_\infty^4 \quad (6)$$

In sight of the equations (3) and (5), the equations (6) becomes

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\left(k+\frac{16\sigma^*T_\infty^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial T}{\partial y}\right)^2 \quad (7)$$

Now by defining the stream function,

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}$$

are satisfies the equation (1). Next, introduce the similarity transformations

$$\psi = U_0 f'(\eta) e^{x/2L}, \quad v = -\sqrt{\frac{\upsilon U_0}{2L}} e^{x/2L}\{f + \eta f'\}$$

$$T = T_0 \theta(\eta) e^{x/2L} \quad \text{where} \quad \eta = y\sqrt{\left(U/2\upsilon L\right)} e^{x/2L}$$

Using the dimensionless and similarity variables, eqns. (2) and (7) reduce *to* the following form:

$$f''' + ff'' - 2(f')^2 = 0 \qquad (8)$$

$$\left(1 + \frac{4}{3}K\right)\theta'' + \Pr(f\theta' - f'\theta) + Ec(f'')^2 = 0 \qquad (9)$$

The boundary conditions becomes in the following form:

$$f = 0, f' = 1, \theta = 1, \quad at\ \eta \to 0$$
$$f' \to 0, \theta \to 0, \quad as\ \eta \to \infty \qquad (10)$$

Where prime denotes diff. with resp. to η.

Where $\Pr = \frac{\mu C_p}{k}$ (Prandtl number),

$$Ec = \frac{U_0^2}{T_0 C_p}\{\text{Eckert number and}$$

$$K = \frac{4\sigma^* T_\infty^3}{k^* k}(\text{Radiation numbers})$$

III. Results and Discussion

The ordinary_differential equations (8) and (9) subject to the boundary conditions (10) is solved numerically using bvp5c MATLAB Package. The results obtained shows the influences of the dimensionless governing parameters, namely radiation parameter, Prandtl number and Eckert number on temperature distribution of the flow and do not affect the value of the wall skin friction coefficient due to the decoupled equations. The taking the different values of a Pr, K and E the heat transfer coefficient— $\theta'(0)$ are presented in the tables 1, 2 and 3. It describes that the heat transfer coefficient increases with an increasing the Prandtl number contradictory increasing the radiation parameter, Eckert number. Comparison with the present results shows a favorable agreement [5] as presented in Table 1, 2 and 3.

Figure 1 demonstrate the effects of Prandtl number, *Pr = 1* Eckert number *Ec = 0.2* and radiation number on the velocity distribution *f(η), f'(η)* are inversely proportional to each other and Temperature distribution *θ(η)*. For all values of *Pr, Ec* and *K*, the velocity profile is unique due to the nonlinear differential equations (8) and (9). Figures 2–4 illuminate the effects of Prandtl number *Pr*, Eckert number *E* and radiation number *K* on the temperature *θ(η)*, respectively. Figure 2 shows the influence of the Prandtl number on the temperature distribution. It is observed from figure that increases in Prandtl number decreases the temperature profile and the thermal boundary layer thickness. Physically, an increasing the values of Prandlt number, thermal diffusivity decreases and these process lead to the decreasing energy capacity that decreases the thermal boundary layer. For further observation, the effects of the Eckert number on the temperature profile and the boundary layer thickness increase somewhat with an increase of Eckert number *Ec* as shown in the figure 3. On the other hand, it has been observed in figure 4 that the temperature profile and the thermal boundary layer thickness increase with an increase of radiation parameter (*K*) For further observation, for a fixed values of *Pr = 1*, the effect of the *Ec* and *K* are illustrated in figure 5. It is seen that the temperature profile increase with an increase of Ec and K and the effect of K are more evident that the effect of Ec.

For a fixed $K = 1$ the effect of the Eckert and Prandtl number on temperature profile are shown in figure 6. It is observed that their effects are reverse, in which the increase in Ec and the decrease in Pr lead to the increase in the temperature.

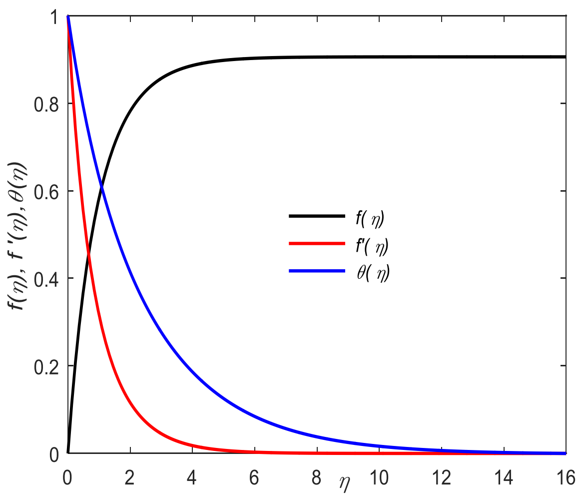

Figure 1: Velocity profile f(η), f'(η) and temperature profile θ(η) for Pr = 1, Ec = 0.2 and K=1.0

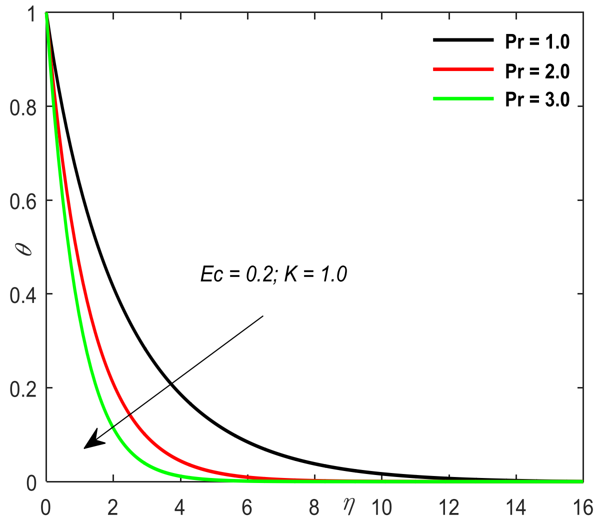

Figure 2: Effects of Pr on the temperature profiles θ(η)

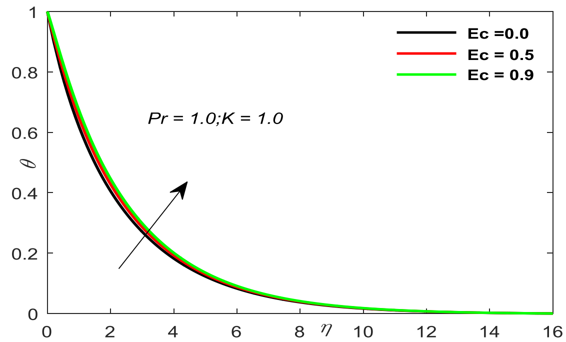

Figure 3: Effects of Ec on the temperature profiles θ(η)

Table 1: Values of the Heat Transfer Coefficient, *-θ'(0)* for Various Values of K and E with Pr = 1, 2, 3

K	E=0			E=0.2			E=0.9		
	Pr = 1	Pr = 2	Pr = 3	Pr =1	Pr =2	Pr = 3	Pr =1	Pr =2	Pr = 3
0	0.954785	1.471463	1.869075	0.862289	1.305534	1.638199	0.538561	0.724793	0.830137
0.5	1.471463	1.073521	1.380755	0.617336	0.965401	1.228688	0.410118	0.586987	0.696467
1	1.869075	0.862774	1.12143	0.487735	0.781818	1.006781	0.334378	0.49848	0.605519

Table 2: Values of the Heat Transfer Factor, *-θ'(0)* for Various Values of Pr and E with K = 0, 0.5, 1.0

K	E=0			E=0.2			E=0.9		
	K = 0	K = 0.5	K = 1.0	K = 0	K = 0.5	K = 1.0	K = 0	K = 0.5	K = 1.0
1	0.954785	0.676542	0.53155	0.862289	0.617336	0.48774	0.538561	0.410118	0.33438
2	1.471463	1.073521	0.86277	1.305534	0.965401	0.78182	0.724793	0.586987	0.49848
3	1.869075	1.380755	1.12143	1.638199	1.228688	1.00678	0.830137	0.696467	0.60552

Table 3: Values of the Heat Transfer Factor, -*θ'(0)* for Several Values of Pr and K with E = 0, 0.2, 0.9

Pr	K = 0			K = 0.5			K = 1		
	Ec = 0	Ec = 0.2	Ec = 0.9	Ec = 0	Ec = 0.2	Ec = 0.9	Ec = 0	Ec = 0.2	Ec = 0.9
1	**0.954785**	**0.862289**	**0.538561**	**0.676542**	**0.617336**	**0.410118**	**0.53155**	**0.48774**	**0.33438**
2	**1.471463**	1.305534	0.724793	1.073521	0.965401	0.586987	0.86277	0.78182	0.49848
3	**1.869075**	1.638199	0.830137	1.380755	1.228688	0.696467	1.12143	1.00678	0.60552

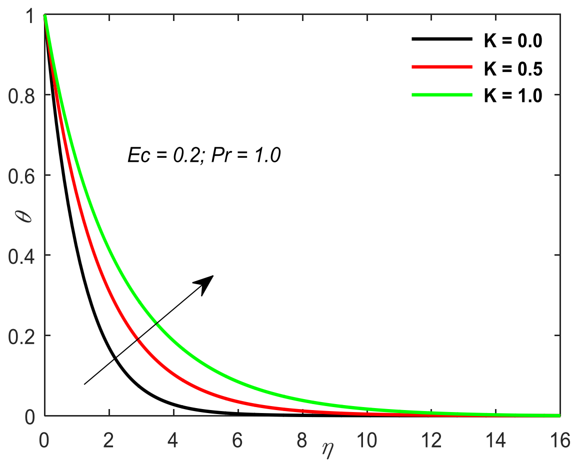

Figure 4: Effects of K on the temperature profiles θ(η)

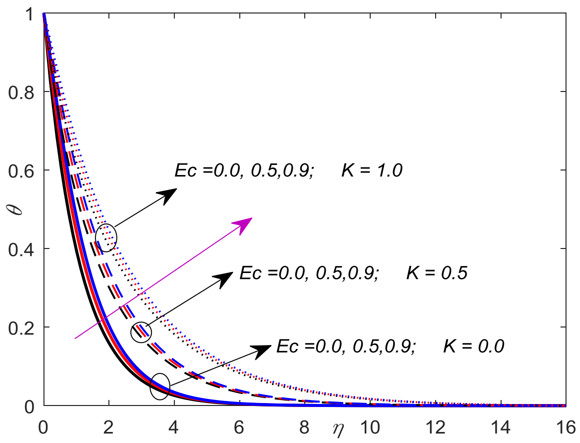

Figure 5: Effects of Ec and K on the temperature profiles θ(η) with Pr = 1

Finally for a fixed *Ec = 0.5*, figure 7 displays the effects of the K and Pr on the temperature profiles. It is evident from figure that although both radiation parameter and Prandtl number have the same effects on the temperature profiles. In difference to the outcomes of Pr, and K are more distinct than the effects of Eckert number.

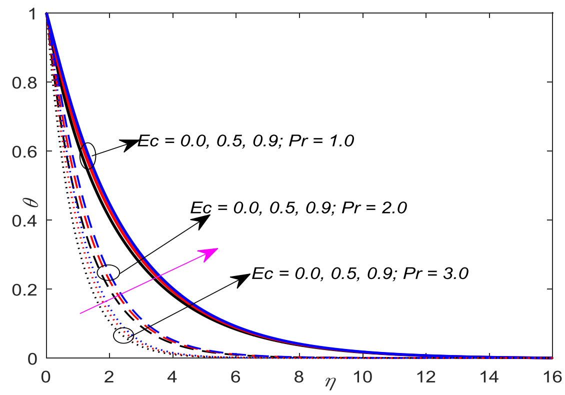

Figure 6: Effects of Ec and Pr on the temperature profiles θ(η) with K =1

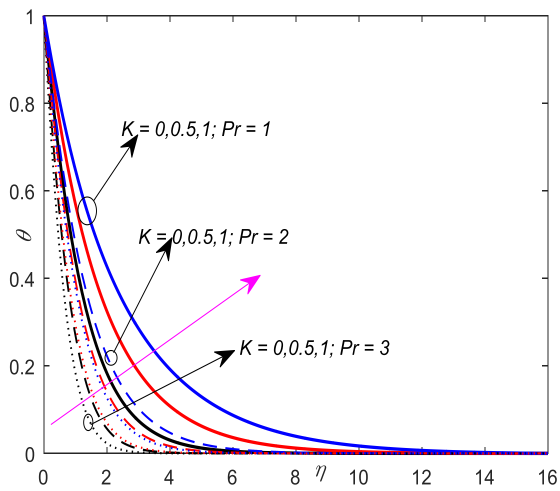

Figure 7: Effects of K and on the temperature profiles with θ(η) with Ec =0.5

V. CONCLUSION

An investigation of numerical study of thermal radiation effect influence on boundary layer flow over an exponentially Stretching Sheet in the presence of viscous dissipation is carried out. Important findings of the problems are mentioned below.

- Temperature of the fluid rising for increasing values for, Ec, K, and (Pr, K), (Ec, Pr), (Ec, K) temperature distribution falling down for increasing the values of Pr.
- Coefficient of Heat transfer increases with an increasing Prandtl number and decreases for the increment of radiation parameter, Eckert number.

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